Coexistence of the "bogolons" and the one-particle spectrum of excitations with a gap in the degenerated Bose gas

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Properties of the weakly non-ideal Bose gas are considered without suggestion on C-number representation of the creation and annihilation operators with zero momentum. The "density-density" correlation function and the one-particle Green function of the degenerated Bose gas are calculated on the basis of the self-consistent Hartree-Fock approximation. It is shown that the spectrum of the one-particle excitations possesses a gap whose value is connected with the density of particles in the "condensate". At the same time, the pole in the "density-density" Green function determines the phonon-roton spectrum of excitations which exactly coincides with one discovered by Bogolyubov for the collective excitations (the "bogolons").

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I. INTRODUCTION

It is well known that theoretical description of the superfluidity phenomenon must take into account the interaction between helium atoms. Also, the closeness of the transition temperature in superfluid state T_λ to the temperature T_0 of Bose condensation for the ideal Bose gas and accumulation of the macroscopic quantity of particles in the state with the momentum zero ("condensate") [1] are the most important features which permit us to apply the models of weakly non-ideal Bose gas to the liquid HeII. Therefore, it seems possible that the phenomenon of superfluidity can be described within the framework of the weakly non-ideal Bose gas model.

It is necessary to stress here that the model of the ideal Bose gas does not satisfy the Landau criterion of superfluidity [2,3] and cannot explain the specific behavior of the thermodynamic properties of the superfluid helium [4]. However, the application of the standard perturbation theory expansion on the inter-particle interaction to the Bose gas at $T < T_0$ at once faces the problem of the appropriate description of the condensate (see, e.g., [5]).

Therefore, the development of the microscopic theory of the degenerated Bose system can be conducted in two ways:

A) The formulation of some special suggestions (conditions) for some functions (operators) for the degenerated Bose gas,

B) Using another initial model instead of the ideal Bose gas for developing the perturbation theory.

Starting with the classical Bogolyubov's papers [6,7], the microscopic theory of the degenerated Bose gas has been based on the special suggestion on the C-number representation of the creation a_0^+ and annihilation a_0 operators of the particles with the momentum $\mathbf{p} = 0$.

The results obtained by Bogolyubov, including the spectrum of the collective excitations (that further be called "bogolons" after the name of the author), permit us to give the qualitative explanation of the experimental data in superfluid helium and to satisfy the Landau condition of superfluidity. It should be mentioned here that the mathematical methods of the quantum field theory were first applied to the study of the non-ideal degenerated Bose gas by Belyaev [8], without special suggestion on the C-number representation of the operators a_0^+ and a_0. In [8], the special diagram technique for the perturbation row at zero temperature, was developed and then generalized for the one- particle Green functions. In particular, it was suggested to consider (apart from the usual Green function with one incoming and, respectively, one outgoing external lines), the additional Green functions with two incoming and two outgoing external lines. However, due to the complexity of its mathematical approach, the method suggested in [8] is not widely used.

Hugenholtz and Pines in [9] reformulated the problem at the beginning, by changing, according to Bogolyubov, the operators a_0^+ and a_0 by C-numbers, and they could use almost automatically the quantum field theory methods for investigating the Bose gas with the "condensate". In particular, in [9], it was shown that for the C-number representation of the operators a_0^+ and a_0 in the excitation spectrum connected with one-particle Green function, the gap cannot exist. Their approach is used presently in the theory of the degenerated Bose gas (see, e.g., [5]).

However, rigorous proof of the correctness of the C-number representation of the operators a_0^+ and a_0 is not possible and, due to this circumstance, the correspondence of the initial Hamiltonian to the Hamiltonian which arises after

changing the operators a_0^+ and a_0 on C-numbers, remains undecided [10,11]. Moreover, in [12-14], the attempts were made to suggest some canonical transformation of the field operators, which would not be connected with the C-number representation of the operators a_0^+ and a_0 (and therefore alternative to the Bogolyubov's one).

Importantly, in [13-16], the possibility of the existence of two different spectra simultaneously - the one-particle one with a gap and the collective one - corresponding to the Bogolyubov's branch of excitations, has been suggested and discussed.

In addition, in [14,18], most of the known results for the thermodynamic properties of the degenerated weakly non-ideal Bose gas have been reproduced by using the "dielectric formalism" without C-number representation of the operators a_0^+ and a_0. Furthermore, in [19], it was argued that the formal restrictions on the use of the standard temperature diagram technique for the temperatures $T < T_0$ are absent.

Another essential circumstance, connected with the C-number representation of the operators a_0^+ and a_0, has to be mentioned. The C-number representation of these operators is based on the commutation relations and the smallness of the parameter $1/N_0$, where N_0 is the operator of the particle number in the state $\mathbf{p} = 0$. Therefore, this parameter makes sense only for the system with fixed number of particles, i.e. in the framework of the canonical ensemble where N_0 is equal to the average $< N_0 >$. Although the Bogolyubov's canonical transformation of the initial Hamiltonian is realized in the canonical ensemble, the calculations of the averages, usually are executed in the grand canonical ensemble.

In the present paper we are exploring the second way, B, for developing the microscopic theory of the degenerated Bose system. We are take into account not only the provision of the Landau superfluidity condition, but also the consecutive description of the weakly non-ideal Bose gas for the transition from temperatures $T \geq T_0$ to the case of $T < T_0$ including the case of a strong degeneration. As an initial approach, we consider the self-consistent Hartree-Fock approximation, which is the best one-particle approximation for the normal systems (see, e.g., [20]).

II. HARTREE-FOCK APPROXIMATION FOR $\delta(\mathbf{r})$ - POTENTIAL AND THE GAP IN THE ONE-PARTICLE SPECTRUM

Let us consider the weakly non-ideal Bose gas which consists of the particles with the mass m and zero spin. The interaction between the particles is described by the potential $U(r)$ with the Fourier-component $u(\mathbf{q})$

$$\lim_{q\to 0} u(q) = u(0) \equiv u(\mathbf{q}=0) > 0 \tag{1}$$

Expression for the one-particle distribution function $f(\mathbf{p})$ reads

$$f(\mathbf{p}) =< a_{\mathbf{p}}^+ a_{\mathbf{p}} >, \quad n = \frac{1}{V}\sum_p f(\mathbf{p}), \tag{2}$$

where $a_{\mathbf{p}}^+$ and $a_{\mathbf{p}}$ are the creation and annihilation operators for the particles with the momentum $\hbar\mathbf{p}$, $n =< N > /V$ is the average density of the particles in the volume V at temperature T, $N = \sum_p a_{\mathbf{p}}^+ a_{\mathbf{p}}$, and the angle brackets $<...>$ denote the averaging in the grand canonical ensemble with the chemical potential μ.

Now let us pay attention to the spectral representation which shows that the function $f(\mathbf{p})$ is not negative for the arbitrary wave vectors $\mathbf{p}$

$$f(\mathbf{p}) \geq 0 \tag{3}$$

In the framework of the self-consistent Hartree-Fock approximation for $T > T_0$, the distribution function $f(\mathbf{p})$ satisfies the relation (see,e.g., [21])

$$f(\mathbf{p}) = \left\{ exp\left(\frac{E(\mathbf{p}) - \mu}{T}\right) - 1\right\}^{-1} \tag{4}$$

where $E(\mathbf{p})$ is the energy of the one-particle excitations

$$E(\mathbf{p}) = \varepsilon(p) + nu(0) + \frac{1}{V}\sum_{\mathbf{q}\neq\mathbf{0}} u(q) f(\mathbf{p}+\mathbf{q}) \tag{5}$$

$\varepsilon(p) = \hbar^2 p^2/2m$ is the energy spectrum of a free particle, the second and the third terms on the right side of (5) correspond to the inter-particle interaction in the Hartree and in the Fock (the so-called exchange interaction, which is conditioned by the identity of the particles) approximations, respectively.

As it will be clear from further consideration, the important point is connected with the presence in (5) of the distribution function $f(\mathbf{p})$ determined by (4), but not the distribution function $f^{id}(\mathbf{p})$ of the ideal gas with spectrum of the free particles, as it is usual in the perturbational Hartree-Fock approximation. Therefore, equations (4),(5) form the closed system of equations for determination of the distribution function and the one-particle excitation spectrum for the fixed thermodynamic parameters. In the language of the temperature diagram technique [5,21], it means the exact summation of some class of the diagrams in the equation for the one-particle Green function.

For the particular case when the interaction potential has the form of the δ-potential

$$U(\mathbf{r}) = u(0)\delta(\mathbf{r}), \quad u(q) = u(0) \tag{6}$$

from (4) and (5) we obtain

$$f(\mathbf{p}) = \left\{ exp\left(\frac{\varepsilon(\mathbf{p}) - \gamma(\mathbf{p}) - \mu^*}{T}\right) - 1 \right\}^{-1} \tag{7}$$

$$\mu^* = \mu - 2nu(0), \quad \gamma(\mathbf{p}) = \frac{u(0)}{V} f(\mathbf{p}) \tag{8}$$

As is follows from Eqs. (7),(8) before transition to the thermodynamic limit, it is necessary to take into account that the system is located in the finite volume V. The importance of this point has been mentioned in [9] in the case of the ideal Bose gas condensation.

Taking into account (8), one can see that Eqs. (7),(8) make sense only for $\mu^* < 0$. As this takes place, the contribution of the function $\gamma(\mathbf{p})$ in (7),(8) is negligible for the arbitrary values of the vector $\mathbf{p}$. In this case, the distribution function $f(\mathbf{p})$ is equivalent to the one for the ideal Bose gas $f^{id}(\mathbf{p})$ with the chemical potential μ changed to μ^*. Correspondingly, in the transition from μ^* to the chemical potential μ^{id}, all known results for the ideal Bose gas are reproduced up to the temperature T_0 of Bose condensation, including the transition temperature itself.

However, the situation changes dramatically for the temperature of Bose condensation T_0, when the accumulation of the particles in the state with the momentum $\mathbf{p} = 0$ starts, and also below this temperature.

By analogy with the case of the ideal Bose gas (see, e.g., [1]) it would seem that one can suppose that

$$\mu^* = 0, \text{ or } \mu = 2nu(0), \tag{9}$$

and that the function $f(\mathbf{p})$ can be represented in the form

$$f(\mathbf{p}) =< N_0 > \delta_{\mathbf{p},0} + f^T(\mathbf{p})(1 - \delta_{\mathbf{p},0}), \tag{10}$$

where $N_0 = a_0^+ a_0$ is the operator of the quantity of particles with the momentum equal zero ("condensate"), $f^T(\mathbf{p})$ is the one-particle distribution function with non-zero momenta (the "overcondensate" states) and

$$< N_0 >=< N > \left\{ 1 - \left(\frac{T}{T_0}\right)^{3/2} \right\}, \quad f^T(\mathbf{p}) = f^T_{id}(\mathbf{p}) = \left\{ exp\left(\frac{\varepsilon(\mathbf{p})}{T}\right) - 1 \right\}^{-1} \tag{11}$$

It is necessary to mention that the relation $f(\mathbf{p}) =< N_0 > \delta_{\mathbf{p},0}$ (see Eq. (10) for $f^T(\mathbf{p}) = 0$) was first suggested in [22].

However, the representation (9) for the chemical potential μ is twice as big as the chemical potential of the ideal Bose gas in the case of strong degeneration. Besides, the function $f^T_{id}(\mathbf{p})$ possesses the singularity at small wave vectors $\mathbf{p}$.

Let us mention now that neglecting the value $\gamma(\mathbf{p})$ at $\mathbf{p} = 0$ for the temperatures $T < T_0$, is not correct for calculating the distribution function $f(\mathbf{p})$ in (7)

$$\gamma(\mathbf{p}) = n_0 u(0), \quad n_0 = \frac{< N_0 >}{V} \tag{12}$$

Therefore, if representation (10) is true, the Eqs. (9),(11) are wrong. Taking into account (10),(12) from Eqs. (7),(8) for $T < T_0$ one directly finds

$$\mu = (2n - n_0)u(0), \tag{13}$$

$$f^T(p) = \left\{ exp\left(\frac{E^*(p)}{T}\right) - 1 \right\}^{-1} (1 - \delta_{\mathbf{p},0}), \quad E^*(p) = \varepsilon(p) + n_0 u(0), \tag{14}$$

$$n_0 = n - \int \frac{d^3p}{(2\pi)^3} f^T(p). \tag{15}$$

On the basis of (13)-(15) it is easy to establish the correctness of the following relations

$$\lim_{T\to 0} n_0 = n, \quad \lim_{T\to 0} \mu = nu(0), \tag{16}$$

$$\Delta = \lim_{p\to 0} E^*(p) = n_0 u(0), \tag{17}$$

$$\lim_{p\to 0} f^T(p) = \left\{ \exp\left(\frac{\Delta}{T}\right) - 1 \right\}^{-1} < \infty, \quad \lim_{T\to 0} f^T(p) = 0. \tag{18}$$

Therefore, in the framework of the self-consistent Hartree-Fock approximation (4),(5) for the one-particle distribution function $f^T(p)$, for the temperatures $T < T_0$ we can establish that:

(A) In the case of strong degeneration, the obtained results (16) coincide with the known relations for the weakly non-ideal Bose gas [5];

(B) In the one-particle excitation spectrum, the gap (17) arises between the "condensate" and the "overcondensate" states. The appearance of the gap (17) is conditioned by the existence of the "condensate". The spectrum of the one-particle excitations, taking into account the "condensate" with the zero energy and the spectrum for the "overcondensate" states, satisfy the Landau condition of superfluidity;

(C) The distribution function for the "overcondensate" states is finite at small values of the wave vectors distinct from the distribution function (11) for the ideal Bose gas. On the basis of the above, it is possible to assert that the self-consistent Hartree-Fock approximation for the one-particle distribution function (4),(5) is suitable as an initial approximation for constructing the theory of Bose gas that takes into account the interaction at arbitrary thermodynamic parameters. It is necessary to mention further that according to (14) in many applications, the condition $T \to 0$ is equivalent to the condition

$$T \ll \Delta. \tag{19}$$

III. COLLECTIVE EXCITATIONS AND THE DIELECTRIC FORMALISM

Having the above results, let us consider the problem of the collective excitations in the weakly non-ideal Bose gas on the basis of the "dielectric formalism"[17,18]. The experimental determination of the collective excitations spectrum is interpreted on the basis of the existing data on the well observed maximums [23] in the dynamical structure factor $S(q,\omega)$ for $q \neq 0$,

$$S(q,\omega) = \frac{1}{V} \int_{-\infty}^{\infty} dt\, exp(i\omega t) < \rho_q(t)\rho_{-q}(0) >, \tag{20}$$

$$\rho_q(t) = \sum_p a^+_{\mathbf{p}-\mathbf{q}/2}(t) a_{\mathbf{p}+\mathbf{q}/2}(t), \tag{21}$$

where $\rho_q(t)$ is the Fourier-component of the operator of particle density in the Heisenberg representation. The dynamical structure factor (20) is directly connected [24] with the retarded density-density Green function $\chi^R(q,z)$ which is analytical in the upper semi-plane of the complex z ($Imz > 0$),

$$S(q,\omega) = -\frac{2\hbar}{1 - exp(-\hbar\omega/T)} Im\chi^R(q,\omega + i0), \tag{22}$$

$$\chi^R(q,z) = -\frac{i}{\hbar V} \int_0^{\infty} dt\, exp(izt) < [\rho_q(t)\rho_{-q}(0)] >>= \frac{1}{V} << \rho_q \mid \rho_{-q} >>_z, \tag{23}$$

The equalities (20),(22) have to be taken in the thermodynamic limit: $V \to \infty$, $< N > \to \infty$ and $< N > /V \to const.$ Eq. (22) serves as the basis for calculation of the function $S(q,\omega)$ for quantum systems by the perturbation methods

of the diagram technique [5,21,24]. The retarded Green function $\chi^R(q,z)$ (23) is the analytical continuation of the temperature Green function $\chi^T(q,i\Omega_n)$

$$\chi^T(q,i\Omega_n)=\frac{1}{V}<<\rho_q\mid\rho_{-q}>>_{i\Omega_n}, \qquad (24)$$

from the discrete multitude of the points on the imaginary axis $i\Omega_n=i2\pi nT$ to the upper semi-plane of the complex z [5,21,24]. For the function $\chi^T(q,i\Omega_n)$, there exists the diagram representation which is connected with the extraction of the irreducible (in q-channel on one line of interaction $u(q)$) part of $\chi^T(q,i\Omega_n)$ - the so-called the polarization operator $\Pi(q,i\Omega_n)$ [21]. After analytical continuation of the function $\chi^T(q,i\Omega_n)$, we arrive at the expression

$$\chi^R(q,z)=\frac{\Pi(q,z)}{\varepsilon(q,z)}. \qquad (25)$$

Here, the function $\varepsilon(q,z)$, by analogy with the terminology accepted in the theory of Coulomb systems [24], is called dielectric permittivity

$$\varepsilon(q,z)=1-u(q)\Pi(q,z). \qquad (26)$$

It should be noted that all relations mentioned above in this section are valid for arbitrary interaction $u(\mathbf{q})$.

Determination of the appropriate approximation for the polarization operator permits us to find the poles of the Green function $\chi^R(q,z)$. These poles describe the collective excitations in the system, which are the solutions of the equation

$$\varepsilon(q,z)=0. \qquad (27)$$

The above equation, meanwhile, is well known from the theory of the Coulomb systems [25].

In considering the case of the weakly non-ideal Bose gas for calculation of the polarization operator $\Pi(q,z)$, we restrict ourselves to the simplest "one-loop" approximation, which in the theory of the Coulomb systems [25] is called "random phase approximation" (RPA). Then, taking into account Eq. (10) one obtains [17,18]

$$\Pi^{RPA}(q,z)=\Pi^{(0)}(q,z)+\Pi^T(q,z). \qquad (28)$$

$$\Pi^{(0)}(q,z)=\frac{2n_0\varepsilon(q)}{\hbar^2z^2-\varepsilon^2(q)}. \qquad (29)$$

and

$$\Pi^T(q,z)=\int\frac{d^3p}{(2\pi)^3}\,\frac{f^T_{id}(\mathbf{p}-\mathbf{q}/2)-f^T_{id}(\mathbf{p}+\mathbf{q}/2)}{\hbar z+\varepsilon(\mathbf{p}-\mathbf{k}/2)-\varepsilon(\mathbf{p}+\mathbf{k}/2)} \qquad (30)$$

Taking into account the above consideration, we now modify the RPA approximation on the generalized (MRPA) approximation, by introducing the change in $\Pi^T(q,z)$ (30) operator. The function f^T_{id} (11) and the spectrum of the one-particle excitations $\varepsilon(p)$ for the ideal Bose gas are changing for the function $f^T(p)$ and the energy $E^*(p)$ (14) in the self-consistent Hartree-Fock approximation. As this takes place, we observed that due to the ruptured character of the one-particle spectrum at the point $\mathbf{p}=0$, the function $\Pi^{(0)}(q,z)$ preserves its earlier form (29). Then

$$\Pi^{MRPA}(q,z)=\Pi^{(0)}(q,z)+\Pi^T_{MRPA}(q,z), \qquad (31)$$

$$\Pi^T_{MRPA}(q,z)=\int\frac{d^3p}{(2\pi)^3}\,\frac{f^T(\mathbf{p}-\mathbf{q}/2)-f^T(\mathbf{p}+\mathbf{q}/2)}{\hbar z+E^*(\mathbf{p}-\mathbf{k}/2)-E^*(\mathbf{p}+\mathbf{k}/2)}. \qquad (32)$$

Considering further the case of a low temperature in (19), one can omit the part $\Pi^T_{MRPA}(q,z)$ (32) in $\Pi^{MRPA}(q,z)$. In this case, from (25)-(27), (29) it directly follows that for the temperatures $T\ll\Delta$

$$\chi^R(q,z)=\frac{2n_0\varepsilon(q)}{\hbar^2z^2-(\hbar\omega(q))^2}, \qquad (33)$$

where the spectrum of the collective excitations is determined by the equality

$$\hbar\omega(q)=\left\{(\varepsilon(q))^2+2\varepsilon(q)nu(q)\right\}^{1/2}. \qquad (34)$$

The relation (34) completely coincides with the spectrum of the "bogolons" for the interaction potential, where its dependence on the wave vector [26,27] provides the fulfilment of the Landau superfluidity condition. Since, in the temperature interval under consideration, the density of the particles in the "codensate" n_0 is close to the total density n, we can change n_0 to n in (34). Then, inserting (33) in (22) and taking into account the determination of the structure factor [1,23], we find [17,18]

$$nS(q) = \frac{\varepsilon(q)}{\hbar\omega(q)}\, cth\left\{\frac{\hbar\omega(q)}{2T}\right\} \tag{35}$$

The above equation is the generalization of the Feinman formula [28] for the connection between the static structure factor and the spectrum

$$\hbar\omega(q) = \varepsilon(q)/S(q), \tag{36}$$

which is valid for the case $\hbar\omega(q) \gg T$.

In the opposite case of $\hbar\omega(q) \ll T$ (and $n_0 \simeq n$, e.g., $T \ll T_0$), we have

$$S(q) = \frac{2\varepsilon(q)T}{\hbar^2\omega^2(q)}, \quad \lim_{q\to 0} S(q) = \frac{T}{nu(0)} \tag{37}$$

Relation (37) corresponds to the general result for the systems with the short-range interaction potential (1) [1]

$$\lim_{q\to 0} S(q) = nTK_T, \quad K_T = -\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_T. \tag{38}$$

Here K_T is the isothermal compressibility of the system. Comparing (37) and (38) and taking into account that the spectrum of the "bogolons" in the region of the small wave vectors has the form

$$\hbar\omega(q) = S_T\, q, \quad S_T = \left(\frac{nu(0)}{m}\right)^{1/2} \tag{39}$$

we conclude that the value S_T characterizes the isothermal sound velocity. In addition, on the basis of Eq. (35) for the static structure factor we can determine the free energy F of weakly non-ideal Bose gas for the temperature $T \ll \Delta$ by using the general relation [1]

$$F = F^{id} + \frac{1}{2}u(0)nN - \frac{1}{2}n\sum_q u(q) + \frac{1}{2}n\sum_q \int_0^1 u(q)S_\lambda(q)d\lambda, \tag{40}$$

$$F^{id} = T\sum_q ln\left[1 - \exp(-\varepsilon(q)/T)\right]. \tag{41}$$

Here F^{id} is the free energy of the ideal Bose gas for $T < T_0$ [26] and $S_\lambda(q)$ is the static structure factor for the system with the interaction potential $\lambda u(q)$. Inserting (35) in (40) we find [17,18]

$$F = \frac{1}{2}u(0)nN - \frac{1}{2}\sum_q\{\varepsilon(q) + nu(q) - \left(\varepsilon(q)^2 + 2\varepsilon(q)nu(q)\right)^{1/2}\} + T\sum_q ln\left\{1 - \exp[-\hbar\omega(q)/T]\right\}. \tag{42}$$

Relation (42) completely corresponds to the results obtained in [5,24,27].

IV. CONCLUSIONS

In the present work we developed the theory of weakly non-ideal Bose gas, below the condensation temperature, on the basis of the self-consistent Hartree-Fock approximation. It was found that the spectra of the collective excitations and the one-particle excitations are distinct, which is contrary to the theory based on the C-number representation for the operators a_0^+ and a_0. It is shown that one-particle branch of excitations has a gap. Both spectra satisfy the Landau criterion of superfluidity. It must be noted that in the early works of Landau, and in the works of Bogolyubov,

they considered the possibility of the existence of the gap in the spectrum of Bose system for $T < T_0$. However, they omitted this suggestion because the phonon branch was absent.

It follows now from the present paper, that the collective phonon-roton spectrum and the one-particle spectrum with a gap actually coexist. On the basis of the calculation of the one-particle distribution function and "density-density" Green function, in the framework of the self-consistent Hartree-Fock approximation for the weakly non-ideal Bose gas, we can establish the following:

1) This system possesses two branches of excitations - the one-particle branch and the collective branch, each of them satisfying the Landau condition of superfluidity.

2) In the region of small wave numbers, there is the gap in the spectrum of the one-particle excitations which is conditioned by the presence of the "condensate".

3) The spectrum of the "bogolons" corresponds to the phonon-roton excitations that are observed in the experiments on the neutron nonelastic collisions [29,30].

Therefore, even a weak inter-particle interaction leads to the drastic differences in the description of the one-particle distribution function and of the excitations of the "overcondensed" particles. We have emphasizes that (as in the case of C-number representation of the operators a_0^+ and a_0) the application of the Hartree-Fock approximation to the calculation of the Green functions for Bose gas in the temperature region $T < T_0$ cannot be absolutely rigorously theoretically justified and there is a need here for further experimental examination. The principal difference between the results of this paper and the traditional approach based on the C-number representation for a_0^+ and a_0, is the appearance of the gap in the spectrum of the one-particle excitations. It follows from the results of this work that the gap cannot manifest itself in the "density-density" Green function (at least in the Hartree-Fock approximation) and therefore it cannot be seen in the experiments on neutron scattering in the superfluid Helium [29,30]. However, such possibility cannot be excluded in the experiments on Raman light scattering. Moreover, in [31], where such experiments are described, there is the direct indication of the existence of the gap.

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